Standing observational issues in ν physics

1930 Pauli postulated the ν existence to save E and $\vec{p}$ conservation in β decay

"I have done a terrible thing proposing a particle that cannot be detected"

1956 $\bar{\nu}_e$ discovery from recators NP 1995
(Savannah River plant) Cowan + Rienes

1962 ν_μ disc. NP Lederman, Schwartz, Steinberger 1988

2000 ν_τ DONUT coll.

2002 NP Davis + Koshiba for detecting solar and SN ν $\rightarrow$ solar ν problem

The discovery of ν masses and mixings through ν oscillations implies a necessary extension of the SM
This extension can be much richer than simply duplicating the quark pattern.
In the ν sector the last "known unknowns" affter the measurement of m_h remain. (and DM sector)
δ (CPV)? mass hierarchy?, neutrino mass? Majorana?
ν masses and mixing imply flavour change in ν propagation $\rightarrow$ ν oscillations

The flavour or interaction eigenstates do not necesarily coincide with the ~~#~~ mass eigenstates

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix} \quad \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix} \quad \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix} \equiv \begin{pmatrix} \nu_\alpha \\ l_\alpha \end{pmatrix}$$

CC $\quad g W_\mu \, \overline{l_\alpha} \, \gamma^\mu P_L \, \nu_\alpha$

$\alpha = e, \mu, \tau \qquad\qquad i = 1, 2, 3$

$U^*_{\alpha i} \, \nu_i$

Matrix to diagonalize the ν mass term

then a ν produced in CC interactions is always an admixture given by U of mass eigenstates

$$|\nu_\alpha\rangle = U_{\alpha i} \, |\nu_i\rangle$$

Or equivalently the Yukawas

$$U = V_\ell^{+L} V_\nu^L$$

$$H \, Y_\ell \, \overline{l}_L \, l_R + \hat{H} \, Y_\nu \, \overline{\nu}_L \, \nu_R$$

U is a unitary matrix $\Rightarrow$ has $\dfrac{N^2 - N}{2}$ angles

$\dfrac{N^2 + N}{2}$ phases

But I can reabsob phases in field redefinitions that leave the Lagrangian ivariant: $\quad l_\alpha \longrightarrow l'_\alpha \, e^{i\theta_\alpha}$

$\nu_j \longrightarrow \nu'_j \, e^{i\theta_j}$

$$g W_\mu \, \overline{l}_\alpha \, \gamma^\mu P_L \, U^*_{\alpha j} \, \nu_j \qquad U^*_{\alpha j} \rightarrow \begin{pmatrix} e^{i(\theta_1 - \theta_e)} & (\theta_2 - \theta_e) & (\theta_3 - \theta_e) \\ (\theta_1 - \theta_\mu) & (\theta_2 - \theta_\mu) & (\theta_3 - \theta_\mu) \\ (\theta_1 - \theta_\tau) & (\theta_2 - \theta_\tau) & (\theta_3 - \theta_\tau) \end{pmatrix}$$

$N + (N-1)$ phases are reabsorbed

$\uparrow$ leptons $\quad \uparrow$ neutrinos

the neutrinos could have a subtlety. Since they are neutral they could have a "Majorana" mass term forbidden for any other fermion

$$m_D\left(\overline{\nu_L}\,\nu_R + \overline{\nu_R}\,\nu_L\right) \qquad m_M\,\overline{\nu_L^c}\,\nu_L$$

Dirac

with $\nu_L^c \equiv i\gamma_0\gamma_2\overline{\nu_L}^t$
$= C\overline{\nu_L}^t$

$\overline{\nu_L^c} = \nu_L C$

$\overline{\nu_L^c}\,\nu_L$ not invariant under phase redefinition $\quad e^{i\theta}\,e^{i\theta}$

If ν are Majorana $\Rightarrow$ $N-1$ phases are physical

For the same reasoon a Majorana mass term breaks any symetry the fermion is charged under $\Rightarrow$
$\Rightarrow$ it breaks lepton number by two units

$\overline{\nu_L^c}\,\nu_L$ can create or destroy $2\,\nu$

If ν where charged it would violate charge conservation

ph. $\frac{N^2+N}{2} - 2N + 1$ $\quad$ a $\frac{N^2-N}{2}$

$\Rightarrow$

Mph $N-1$

	angles	phases	"Majorana phases"
$N=2$	1	0	1
$N=3$	3	1	2

phases violate CP sym that relates particles and antiparticles:

$$\frac{g}{\cancel{\sqrt{2}}} W^+_\mu \bar{\nu}_j \gamma^\mu P_L U^*_{\alpha j} l_\alpha + \frac{g}{\cancel{\sqrt{2}}} W^-_\mu \bar{l}_\alpha \gamma^\mu P_L U_{\alpha j}\nu_j$$

$$\Big\downarrow C$$

$$g W^-_\mu \bar{l}_\alpha \gamma^\mu P_R U^*_{\alpha j} \nu_j + g W^+_\mu \bar{\nu}_j \gamma^\mu P_R U_{\alpha j} l_\alpha$$

$$\Big\downarrow P$$

$$g W^-_\mu \bar{l}_\alpha \gamma^\mu P_L U^*_{\alpha j} \nu_j + g W^+_\mu \bar{\nu}_j \gamma^\mu P_L U_{\alpha j} l_\alpha$$

C and P are completely violated by weak interactions (they would couple the righ-handed chirality of fermions) but CP is <u>only</u> violated if U has <u>imaginary</u> <u>terms</u>

And since $|\nu_\alpha\rangle = U_{\alpha i}|\nu_i\rangle$ $\boxed{\left(\nu_\alpha = U^*_{\alpha i}\nu_i \Rightarrow |\nu_\alpha\rangle = U_{\alpha i}|\nu_i\rangle\right)}$

$|\bar{\nu}_\alpha\rangle = U^*_{\alpha i}|\nu_i\rangle$

the most well-known consequence of ν masses and mixing is ν oscillation (where it was discovered).

In 2 families: $\alpha = e, \mu$ $\quad i = 1, 2$

$$U_{\alpha i} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

$|\nu_e\rangle = \cos\theta\,|\nu_1\rangle + \sin\theta\,|\nu_2\rangle$

$|\nu_\mu\rangle = -\sin\theta\,|\nu_1\rangle + \cos\theta\,|\nu_2\rangle$

$\cos\theta \equiv c \qquad \sin\theta \equiv s$

I always produce or detect a ν_e or ν_μ in CC

But the mass eigenstates are eigenstates of the free $\mathcal{H}$

$$H|\nu_i\rangle = E_i|\nu_i\rangle = \sqrt{p^2 + m_i^2}\,|\nu_i\rangle$$

equal momentum aprox.

Imagine I produce a ν_μ from $\pi^+ \to \mu^+ \nu_\mu$

if I try to detect it as a ν_e

$$\langle\nu_e|\nu_\mu\rangle = -sc\,\langle\nu_1|\nu_1\rangle + sc\,\langle\nu_2|\nu_2\rangle = 0$$

But if I let it propagate a distance (baseline) L:

$$|\nu_\mu(L)\rangle = e^{-iE_1 t}|\nu_1\rangle + e^{-iE_2 t}|\nu_2\rangle$$

with $t \simeq L$ and $E_i \doteq \sqrt{p^2 + m_i^2} \simeq p + \frac{m_i^2}{2p} \simeq p + m_i^2/2E$

$$\Rightarrow \langle\nu_e|\nu_\mu(L)\rangle = sc\,e^{-ipL}\left\{-e^{-im_1^2 L/2E} + e^{-im_2^2 L/2E}\right\}$$

$$|\;\;|^2 = 2 - 2\cos\left(\frac{\Delta m^2 L}{2E}\right)$$

$$\boxed{P_{\nu_\mu\to\nu_e}(L) = |\langle\nu_e|\nu_\mu(L)\rangle|^2 = \sin^2 2\theta\,\sin^2\left(\frac{\Delta m^2 L}{2E}\right)}$$

Alternative derivation: $|\nu_\alpha(t,L)\rangle = U_{\alpha i}\, e^{-i(p^0 t - \vec{p}\,\vec{L})}\, |\nu_i\rangle$

$$\langle \nu_\beta | \nu_\alpha(t,L)\rangle = U^*_{\beta j} U_{\alpha i}\, e^{-i(E_i t - p_i L)} \langle \nu_j | \nu_i \rangle$$

$$t \simeq L \qquad -E_i t + p_i L = (p_i - E_i) L =$$

$$= -\frac{E_i^2 - p_i^2}{E_i + p_i} L = \frac{-m_i^2}{E_i + p_i} L \simeq -\frac{m_i^2 L}{2E}$$

This was using plane wave aprox (which is inconsistent) but with a more cumbersome wave packet formalism one reches the same conclusion $\Rightarrow$ $\Delta E + \Delta p$ needed for oscilletion coherence

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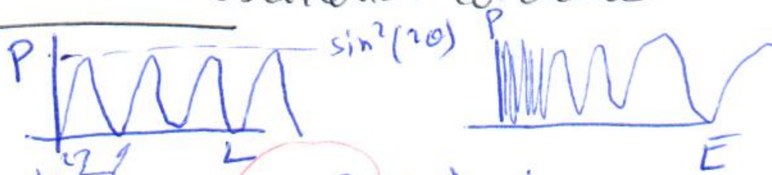
In 2 families in vacuum



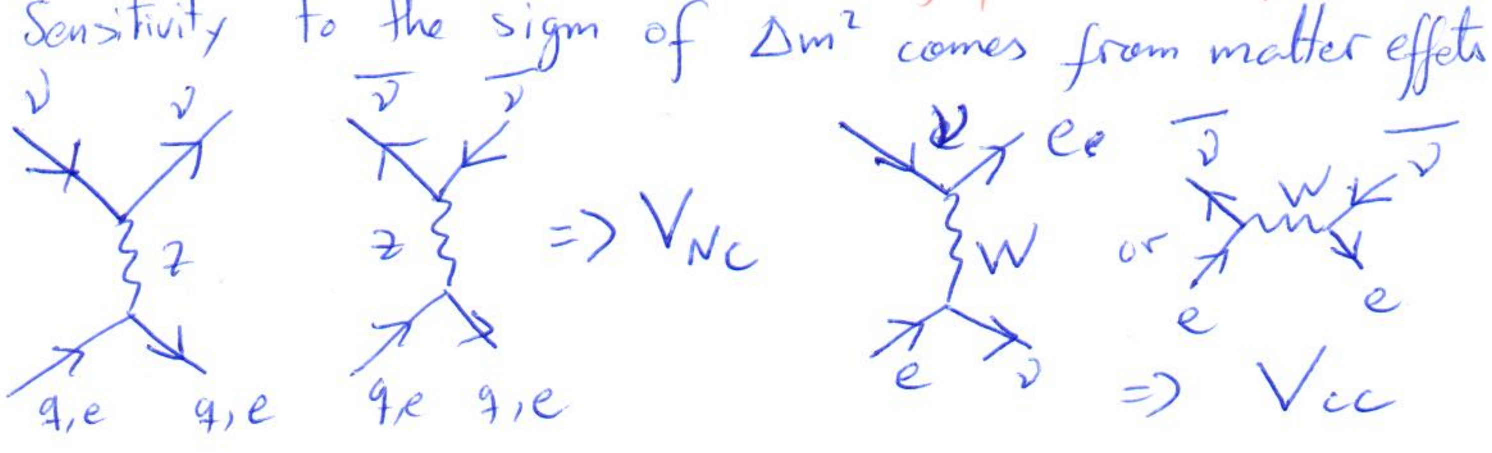
$$P_{\alpha\beta}(L) = \left| \delta_{\alpha\beta} - \sin^2 2\theta \, \sin^2\left(\frac{\Delta m^2 L}{4E}\right) \right|$$

That is why they are called oscillations

Cannot determine sign

$\Rightarrow$ can "flip" the ν spectrum

Sensitivity to the sign of Δm^2 comes from matter effets



The Schrödinger eq. will be modified:

$$i\frac{d}{dt}\begin{pmatrix}|\nu_e\rangle \\ |\nu_\mu\rangle\end{pmatrix} = H\begin{pmatrix}|\nu_e\rangle \\ |\nu_\mu\rangle\end{pmatrix} = \left\{ \overset{U}{\begin{pmatrix} c & s \\ -s & c\end{pmatrix}} \begin{pmatrix} p + \frac{m_1^2}{2E} & 0 \\ 0 & p + \frac{m_2^2}{2E}\end{pmatrix} \overset{U^+}{\begin{pmatrix} c & -s \\ s & c\end{pmatrix}} + \begin{pmatrix} V_{CC} + V_{NC} & 0 \\ 0 & V_{NC}\end{pmatrix} \right\} \begin{pmatrix}|\nu_e\rangle \\ |\nu_\mu\rangle\end{pmatrix}$$

$$H = \underbrace{\begin{pmatrix} p + \frac{m_1^2+m_2^2}{4E} + V_{NC} & 0 \\ 0 & p + \frac{m_1^2+m_2^2}{4E} + V_{NC}\end{pmatrix}}_{\text{global phase} \Rightarrow \text{will cancel when computing } |\ \ |^2} + \begin{pmatrix} c & s \\ -s & c\end{pmatrix} \begin{pmatrix} \overbrace{\frac{m_1^2-m_2^2}{4E}}^{\Delta} & 0 \\ 0 & \frac{m_2^2-m_1^2}{4E} = -\Delta\end{pmatrix} \begin{pmatrix} c & s \\ s & c\end{pmatrix} + \begin{pmatrix} V_{CC} & 0 \\ 0 & 0\end{pmatrix}$$

$$\Rightarrow i\frac{d}{dt}\begin{pmatrix}|\nu_e\rangle \\ |\nu_\mu\rangle\end{pmatrix} = \left\{ \begin{pmatrix} -\cos 2\theta & \sin 2\theta \\ \sin 2\theta & \cos 2\theta\end{pmatrix}\Delta + \begin{pmatrix} V_{CC} & 0 \\ 0 & 0\end{pmatrix}\right\}\begin{pmatrix}|\nu_e\rangle \\ |\nu_\mu\rangle\end{pmatrix}$$

$$V_{CC} = \sqrt{2}\, G_F\, \underbrace{n_e(t)}_{\text{electron density}}$$

depending on matter profile eq can be very difficult to solve

2 simple and useful cases:

1. n_e constant   can be reasonable aprox in Earth

Just diagonalize $H = \Delta\begin{pmatrix} -\cos 2\theta & \sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{pmatrix} + \begin{pmatrix} V_{cc} & 0 \\ 0 & 0 \end{pmatrix}$

new eigenvalues m_m and eigenstates $\to \begin{pmatrix} \cos\theta_m & \sin\theta_m \\ -\sin\theta_m & \cos\theta_m \end{pmatrix}$

$$\sin^2 2\theta_m = \frac{\sin^2 2\theta}{\sin^2 2\theta + \left(\cos 2\theta - \frac{V_{cc}}{2\Delta}\right)^2}$$

$$\Delta m_m^2 = \Delta m^2 \sqrt{\sin^2 2\theta + \left(\cos 2\theta - \frac{V_{cc}}{2\Delta}\right)^2}$$

and $P_{\alpha\to\beta} = \sin^2 2\theta_m \sin^2\left(\frac{\Delta m_m^2 L}{4E}\right)$

If $\frac{V_{cc}}{2\Delta} = \frac{2\sqrt{2}\, G_F n_e E}{\Delta m^2} = \cos 2\theta$

$\Rightarrow \sin^2 2\theta_m = 1 \Rightarrow$ maximal oscillation even if $\theta \ll 1$

Mikeev + Smirnov + Wolfenstein resonance

if $\Delta m^2 > 0$ and $V > 0$ (ν) resonance

if $\Delta m^2 < 0$ an $V < 0$ $(\bar{\nu})$ "

If $\overset{(-)}{\nu}$ probability is enhanced over $\bar{\nu} \Rightarrow \Delta m^2 > 0$

" $\bar{\nu}$ " " " " $\nu \Rightarrow \Delta m^2 < 0$

2. $n_e(t)$ changes very slowly (adiabatically)

this is a good aprox for the Sun matter profile $\quad |\dot{V}| \dfrac{\sin 2\theta \frac{\Delta m^2}{2E}}{|E_2 - E_1|^2} \ll |E_2 - E_1|$

In this case the neutrino has time to "adapt" to the chang of the potential and the solution at a given time t

is $\quad |\nu(t=0)\rangle = \alpha |\nu_1(t=0)\rangle + \beta |\nu_2(t=0)\rangle$

with $\quad H(t)|\nu_i(t)\rangle = E_i(t)\,|\nu_i(t)\rangle$

eigenstates

$$\Rightarrow \boxed{|\nu(t)\rangle = \alpha\,|\nu_1(t)\rangle + \beta\,|\nu_2(t)\rangle}$$

In the center of the sun ν_e are produced

and $n_e \uparrow\uparrow \Rightarrow V_{cc} \gg \Delta$

$$H(t=0) \simeq \begin{pmatrix} V_{cc} & 0 \\ 0 & 0 \end{pmatrix} \Rightarrow \nu_e \text{ is an eigenstate of } H!!$$

$\Rightarrow \nu_e(t\uparrow\uparrow)$ out of the sun must also be an eigenstate

Since ν_e is the ν with largest eigen~~state~~ value

$\Rightarrow$ ν_e emerges as ν_2 $\to$ largest eigenvalue in vacuum

$$|\nu_e\rangle \xrightarrow[\text{conversion}]{\text{adiabatic}} |\nu_2\rangle$$

Sun produces $|\nu_2\rangle$!

Not really an oscillation

Now for 3 families: $|\nu_\alpha\rangle = U_{\alpha i} |\nu_i\rangle$

$$|\nu_\alpha(t)\rangle = e^{-ipL} e^{-i m_i^2 L/2E} U_{\alpha i} |\nu_i\rangle$$

$$\langle \nu_\beta | \nu_\alpha(L)\rangle = e^{-ipL} e^{-i\frac{m_i^2 L}{2E}} U^*_{\beta i} U_{\alpha i}$$

$$\boxed{P_{\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\beta}(L) = |\langle \nu_\beta | \nu_\alpha(L)\rangle|^2} = \sum_{i,j} U_{\alpha i} U^*_{\beta i} U^*_{\alpha j} U_{\beta j} \, e^{i\frac{\Delta m^2_{ij} L}{2E}} =$$

$$= \delta_{\alpha\beta} - 4 \sum_{i>j} \mathrm{Re}\left(U_{\alpha i} U^*_{\beta i} U^*_{\alpha j} U_{\beta j}\right) \sin^2\left(\frac{\Delta m^2_{ij} L}{4E}\right)$$

$$\pm 2 \sum_{i>j} \underbrace{\mathrm{Im}\left(U_{\alpha i} U^*_{\beta i} U^*_{\alpha j} U_{\beta j}\right)}_{\text{Jarlskog invariant}} \sin\left(\frac{\Delta m^2_{ij} L}{2E}\right)$$

CP violation

"GIM" cancellation

$\alpha = \beta$, $i \neq j$ $\Rightarrow$ $J = \frac{1}{8} \sin 2\theta_{12} \sin 2\theta_{23} \sin 2\theta_{13} \cos\theta_{13} \sin\delta$

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} e^{-i\delta} \\ 0 & 1 & 0 \\ s_{13} e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & & \\ & e^{i\alpha_2} & \\ & & e^{i\alpha_3} \end{pmatrix}$$

θ_{23} "atmospheric" mixing angle

$\theta_{23} \simeq 45°$

θ_{13} "reactor" mixing angle

$\theta_{13} \simeq 8.5°$

δ CP fase = ?

θ_{12} "solar" angle

$\theta_{12} \simeq 33°$

α_2, α_3 Majorana phases

do not participate in oscillations

$$\det\left[Y_u Y_u^\dagger, Y_d Y_d^\dagger\right] \frac{v^{12}}{2^6} = -2F(m_u^2)F(m_d^2) J$$

$$F(m_u) = (m_t - m_u)^2 (m_u - m_c)^2 (m_c - m_t^2)$$

$$J = \frac{1}{8} \sin 2\theta_{12} \sin 2\theta_{23} \sin 2\theta_{13} \cos\theta_{13} \sin\delta$$

$J_{quarks} = 2.9 \cdot 10^{-5}$

$J_\nu \simeq 0.035 \sin\delta$

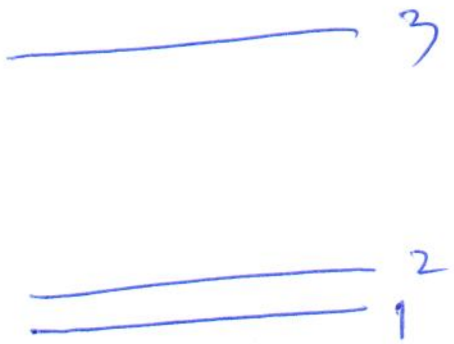
Measure of CPV in U

Only complex invariant

2 mass splittings

$\Delta m^2_{21} \simeq 7.5 \, 10^{-5} \, eV^2$ "solar spliting" → Know sign!

$|\Delta m^2_{31}| \simeq 2.4 \cdot 10^{-3} \, eV^2$ "atmospheric splitting"



"Normal" hierarchy

"inverted" hierarchy

Because $\theta_{13} << \theta_{12}, \theta_{23}$ and $\Delta m^2_{21} << |\Delta m^2_{31}|$

Simple 2-family aprox are recovered in almost all regimes:

• "Solar regime" $E \sim$ few MeV

$\nu_e \longrightarrow \nu_2$ from sun too low E to detect as ν_μ or ν_τ

$|U_{e3}| \simeq \sin\theta_{13}$ too small for ν_3

$$\Rightarrow P^{sun}_{e\to e} \simeq |\langle \nu_e | \nu_2 \rangle|^2 = |U_{e2}|^2 \rightarrow \theta_{12}$$

and knowledge that $m_2 > m_1$

SNO, SK, Borexino

Confirmed by Kland $E \sim$ few MeV $L \simeq 100$ Km

with $U_{e3} \simeq 0 \Rightarrow P^{Kland}_{e\to e} = \sin^2 2\theta_{12} \sin^2\left(\frac{\Delta m^2_{21} L}{4E}\right)$

• "Atmospheric regime" $E \sim$ few GeV $L \sim 1000$ Km

SK with $\frac{\Delta m^2_{21} L}{4E} \simeq 0 << \frac{\Delta m^2_{31} L}{4E} \simeq \frac{\Delta m^2_{32} L}{4E}$

confimed by K2K MINOS T2K

$$P_{\nu_\mu \to \nu_\tau} \simeq \sin^2 2\theta_{23} \sin^2\left(\frac{\Delta m^2_{31} L}{4E}\right) \quad |\Delta m^2_{31}|$$

$P_{\nu_\mu \to \nu_e} \simeq 0$ | $P_{\nu_\mu \to \nu_\mu} = 1 - P_{\nu_\mu \to \nu_\tau}$ measured

$\Leftarrow U_{e3} \simeq 0$ and no $1 \leftrightarrow 2$ oscillations

- "Reactor Regime" $E \sim$ few MeV $L \sim 1$ Km

$$P_{\bar{\nu}_e \to \bar{\nu}_e} \simeq \sin^2 2\theta_{13} \sin^2\left(\frac{\Delta m^2_{31} L}{4E}\right)$$

Daya Bay + RENO (double CHOOZ)

- Accelerator searches for CPV and hierarchy

T2K + NOνA

$$P_{\nu_\mu \to \nu_e} \simeq s^2_{23} \sin^2 2\theta_{13} \sin^2\left(\frac{\Delta m^2_{31} L}{4E}\right) +$$

"atmo"

"solar" $$+ c^2_{23} \sin^2 2\theta_{12} \sin^2\left(\frac{\Delta m^2_{21} L}{4E}\right)$$

"CP" interference $$+ c_{13} \sin 2\theta_{12} \sin 2\theta_{23} \sin 2\theta_{13} \cos\left(\pm\delta - \frac{\Delta m^2_{31} L}{4E}\right) \sin\left(\frac{\Delta m^2_{31} L}{4E}\right) \sin\left(\frac{\Delta m^2_{21} L}{4E}\right)$$

$\sim J$

T2K $L = 295$ Km $E \sim 1$ GeV

NOνA $L = 810$ Km $E \sim 3$ GeV

will start providing first tests

Oscillations cannot probe absolute ν mass scale only Δm^2

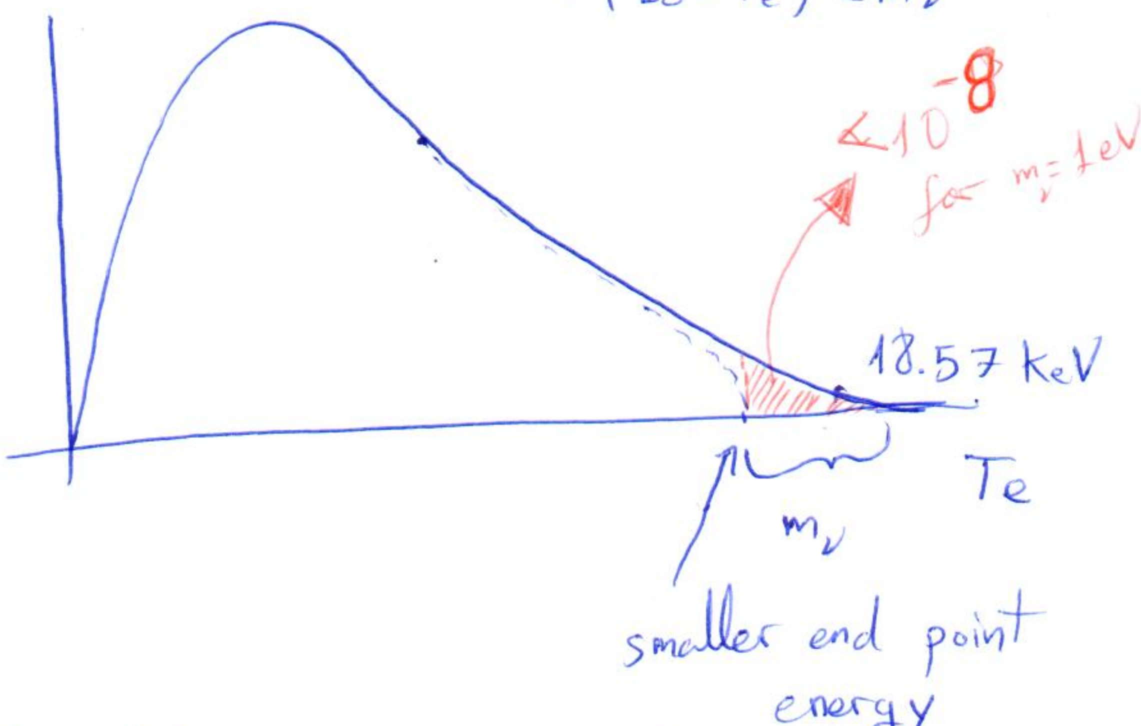
· Direct kinematic searches

Tritium β decay (low $E \Rightarrow$ make more evident ν mass)

$$E_0 = 18.57 \text{ KeV} \qquad T \to {}^3He + e + \bar{\nu}_e$$

$$\frac{dN}{dT_e} \propto |p_e| E_e |p_\nu| E_\nu \overset{\text{no. rel.}}{=} \sqrt{2 T_e m_e}\,(m_e + T_e)(E_0 - T_e) \cdot \sqrt{(E_0 - T_e)^2 - m_\nu^2}$$

e^- Kinetic E



$$\text{"}m_{\nu_e}\text{"} \equiv \sum_i |U_{ei}|^2 m_i < 2.3 \text{ eV} \quad \text{(Mainz + Troitsk)}$$

KATRIN $\rightarrow$ < 0.2 eV