How Could Neutrinos Have Mass?

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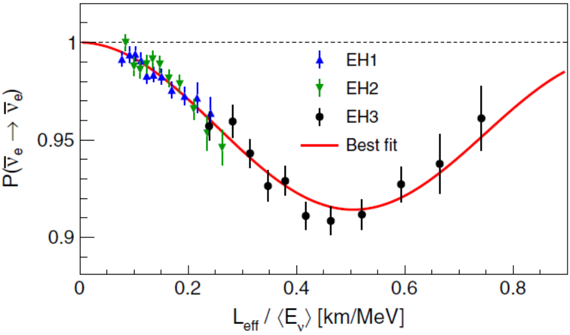
Oscillating Neutrinos Need Mass

- Neutrinos oscillate among flavors:

$$P(\overline{\nu}_e \to \overline{\nu}_e) = 1 - \sin^2 2\theta \sin^2 \frac{\Delta m^2 L}{4E}, \quad \Delta m^2 \equiv m_{\nu_2}^2 - m_{\nu_1}^2$$

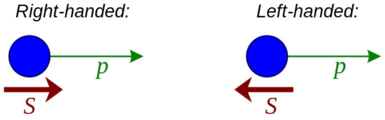
- Oscillatory behavior observed by Daya Bay $\overline{\nu}_e$, KamLand $\overline{\nu}_e$ and SuperKamiokande atmospheric ν_μ data



Daya Bay (2015)

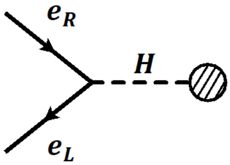
How do Neutrinos Gain Mass?

- Neutrinos are Spin-1/2 Fermions which have an intrinsic property called Helicity:



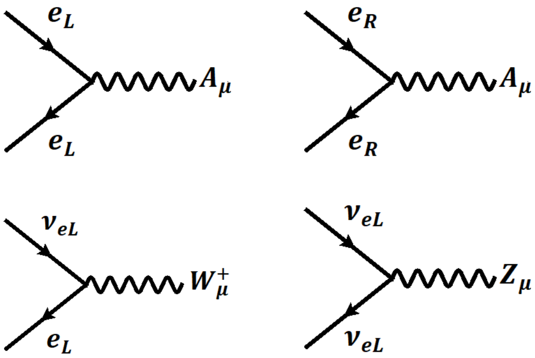
- A related concept for massive fermions is Chirality. A mass term flips left-handed fermion into a right-handed fermion and vice versa.

- In the Standard Model such chirality flips arise through couplings to the Higgs boson:



How do Neutrinos Gain Mass? (cont.)

- The couplings of electrons to the photon does not change chirality:



- Neutrinos appear in nature only as left-handed. Mass generation via Higgs mechanism is not so trivial. In fact, in the Standard Model, neutrinos are exactly massless! New physics is called for.

- Neutrinos we know of are all left-handed (ν_L); while the anti-neutrinos are all right-handed $(\nu^c)_R$

Neutrinos Gaining Mass

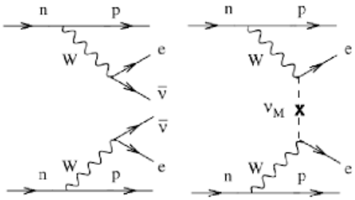
- Every fermion has its anti-fermion partner with opposite charge: electron e^- $\leftrightarrow$ positron e^+. Positron has positive charge.
- Antiparticle of e_L^- is e_R^+ – chirality flips between particle and antiparticle
- Since neutrinos are neutral, a chirality flipping mass term between ν_L and $(\nu^c)_R$ is possible. Electric charge conservation forbids such a mass for the electron, and for all charged fermions
- If this is the source of its mass, neutrino will be a Majorana particle. That is, neutrino is its own antiparticle. All other spin-$1/2$ particles are Dirac fermions.
- Majorana mass is only possible for the neutrino among all elementary fermions. Majoran neutrinos would imply that Lepton Number is a broken symmetry.

Neutrinoless Double Beta Decay

- Lepton number is an accidental symmetry of the Standard Model. Electrons and neutrinos have $L = 1$, while positrons, antineutrinos have $L = -1$. A Majorana mass term breaks L by two units.

- Neutrino oscillation experiments are blind to the Dirac nature or the Majorana nature of the neutrino mass.

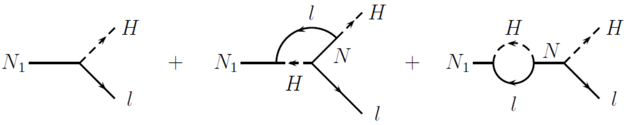
- Double beta decay is a rare process where neutron converts itself into proton + electron and an anti-neutrino twice inside a nucleus:



- If neutrino has a Majorana mass, even rarer neutrino-less double beta decay can occur.

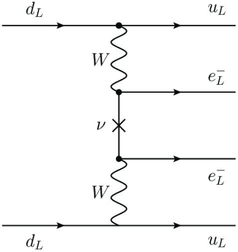
Neutrinoless Double Beta Decay (cont.)

- Observation of neutrino-less double beta decay would confirm lepton number violation by two units, and one can infer the Majorana nature of neutrino.
- This process plays a crucial role in the idea of Leptogenesis, that creates the baryon asymmetry of the Universe.



- Here a Majorana fermion N_1 decays into lepton plus Higgs ($N_1 \to \ell + H$); the same N_1 also decays into antilepton plus anti-Higgs ($N_1 \to \overline{\ell} + \overline{H}$)

Neutrinoless Double Beta Decay Limit on m_ν



$$(A, Z) \to (A, Z+2) + 2e^- + Q_{\beta\beta}$$

- Majorana vs Dirac neutrinos: Observation of $\beta\beta 0\nu$ will establish neutrinos are Majorana particles
- Kamland-Zen collaboration has a limit from ^{136}Xe:

$$T_{0\nu}^{1/2} > 1.07 \times 10^{26}\ \mathrm{yr.}$$

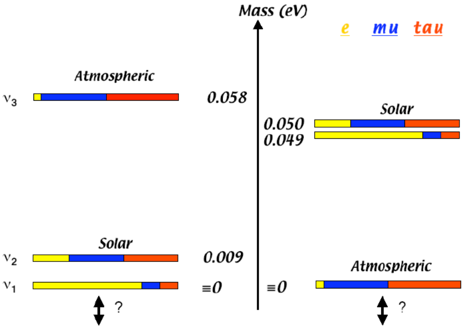
- Constrains effective double beta decay mass of neutrino to be

$$m_{\beta\beta} < (61 - 165)\ \mathrm{meV}$$

$$m_{\beta\beta} = |\sum_i U_{ei}^2 m_i| = |c_{12}^2 c_{13}^2 e^{2i\alpha_1} m_1 + c_{13}^2 s_{12}^2 e^{2i\alpha_2} m_2 + s_{13}^2 m_3|$$

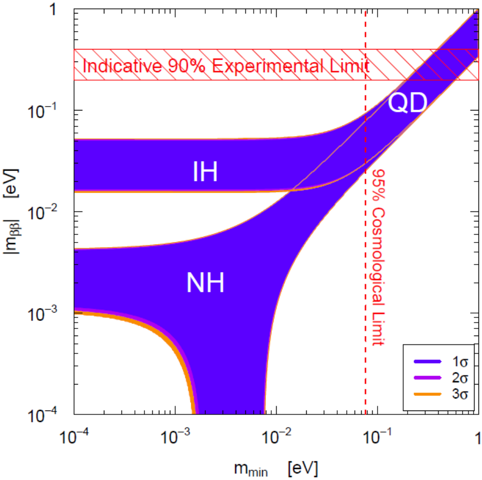
Neutrino Mass Ordering

- Current data allows for two possible ordering of the neutrino masses:



- If mass ordering is normal: $0 \leq m_{\beta\beta} \leq 4\ \mathrm{meV}$
- If mass ordering is inverted: $20\ \mathrm{meV} \leq m_{\beta\beta} \leq\ 50\ \mathrm{meV}$

Experimental Limit on $m_{\beta\beta}$ versus $m_{\min}$

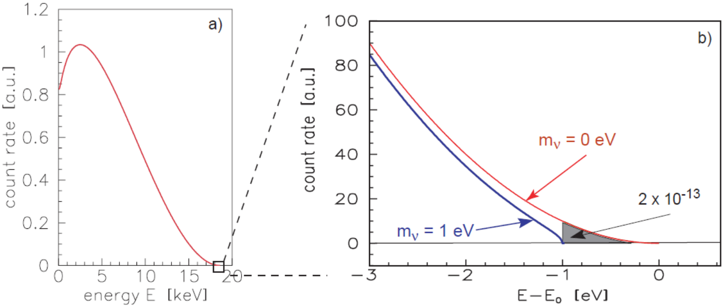


Bilenky, Giunti (2014)

Absolute Neutrino Mass

- Neutrino oscillation experiments only sensitive to $\Delta m_{ij}^2 = m_{\nu_i}^2 - m_{\nu_j}^2$. Absolute scale of neutrino mass is left undetermined.
- Beta decay sepctrum near the end point of electron energy can test directly neutrino mass. Katrin experiment has the best limit from ^3H beta decay:

$$m_\nu \leq 0.8 \text{ eV}$$



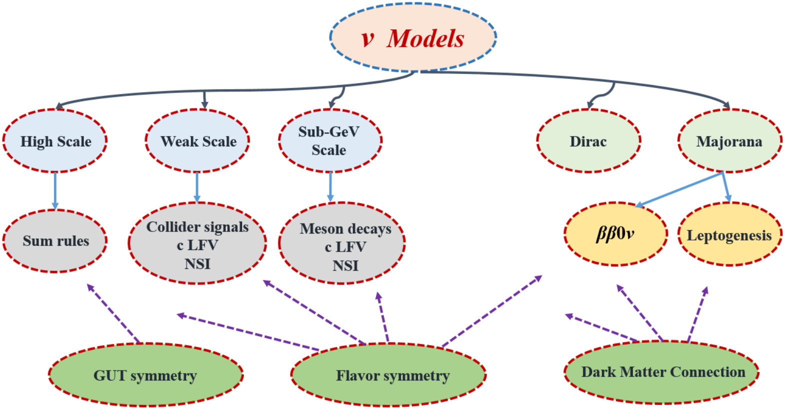
Current knowledge of 3-neutrino oscillations

NuFIT 5.1 (2021)

without SK atmospheric data		Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 2.6$)	
		bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
	$\sin^2\theta_{12}$	$0.304^{+0.013}_{-0.012}$	$0.269 \to 0.343$	$0.304^{+0.012}_{-0.012}$	$0.269 \to 0.343$
	$\theta_{12}/^\circ$	$33.44^{+0.77}_{-0.74}$	$31.27 \to 35.86$	$33.45^{+0.77}_{-0.74}$	$31.27 \to 35.87$
	$\sin^2\theta_{23}$	$0.573^{+0.018}_{-0.023}$	$0.405 \to 0.620$	$0.578^{+0.017}_{-0.021}$	$0.410 \to 0.623$
	$\theta_{23}/^\circ$	$49.2^{+1.0}_{-1.3}$	$39.5 \to 52.0$	$49.5^{+1.0}_{-1.2}$	$39.8 \to 52.1$
	$\sin^2\theta_{13}$	$0.02220^{+0.00068}_{-0.00062}$	$0.02034 \to 0.02430$	$0.02238^{+0.00064}_{-0.00062}$	$0.02053 \to 0.02434$
	$\theta_{13}/^\circ$	$8.57^{+0.13}_{-0.12}$	$8.20 \to 8.97$	$8.60^{+0.12}_{-0.12}$	$8.24 \to 8.98$
	$\delta_{\mathrm{CP}}/^\circ$	194^{+52}_{-25}	$105 \to 405$	287^{+27}_{-32}	$192 \to 361$
	$\frac{\Delta m^2_{21}}{10^{-5}\ \mathrm{eV}^2}$	$7.42^{+0.21}_{-0.20}$	$6.82 \to 8.04$	$7.42^{+0.21}_{-0.20}$	$6.82 \to 8.04$
	$\frac{\Delta m^2_{3\ell}}{10^{-3}\ \mathrm{eV}^2}$	$+2.515^{+0.028}_{-0.028}$	$+2.431 \to +2.599$	$-2.498^{+0.028}_{-0.029}$	$-2.584 \to -2.413$

Esteban, Gonzalez-Garcia, Maltoni, Schwetz, Zhou (2020)

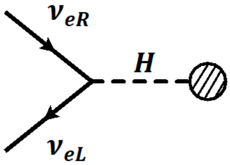
Roadmap for Neutrino Models



Giving the Neutrino a Mass

- Neutrino masses are zero in the Standard Model, since right-handed neutrino is not present

- Neutrino mass can be generated by adding right-handed neutrinos:

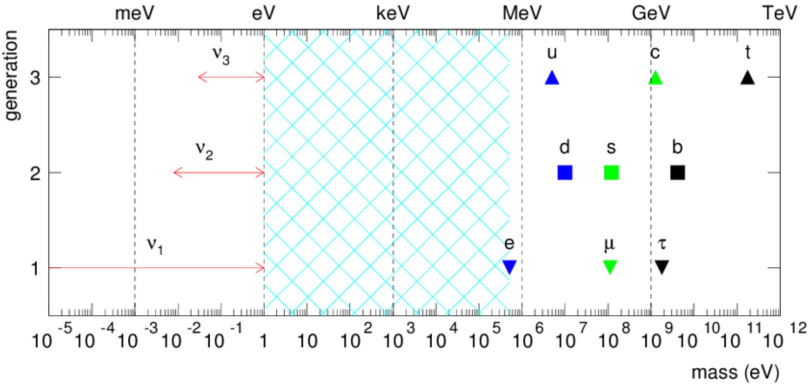


- Neutrino has now a Dirac mass which can explain all oscillation data

- However, the coupling of ν_R with the Higgs boson has to be extremely tine, $\sim 10^{-13}$, which is viewed as unnatural. (Natural couplings are of order one!)

- Such a Dirac neutrino will exasperate the fermion mass hierarchy puzzle

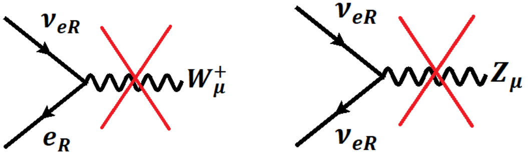
Fermion Mass Hierarchy Puzzle



- The big gap between neutrino masses and charged fermion masses will be unexplained in case of Dirac neutrinos

Features of the putative right-handed neutrino

- ν_R, if it exists, has no weak interactions. It is a sterile component of the neutrino

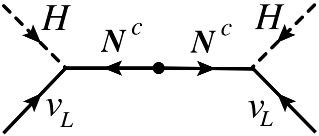


- ν_R only takes part in the neutrino mass generation mechanism
- In fact, one can write down a Majorana mass for the ν_R. This mass can be very large, much larger than the electroweak scale
- In this case one would realize the seesaw mechanism for naturally small neutrino masses!

Origin of neutrino mass: Seesaw mechanism

- Adding right-handed neutrino N^c which transforms as singlet under $SU(2)_L$,

$$\mathcal{L} = f_\nu \, (L \cdot H) \, N^c + \frac{1}{2} M_R N^c N^c$$



- Integrating out the N^c, $\Delta L = 2$ operator is induced:

$$\mathcal{L}_{\text{eff}} = -\frac{f_\nu^2}{2} \frac{(L \cdot H)\,(L \cdot H)}{M_R}$$

- Once H acquires VEV, neutrino mass is induced:

$$m_\nu \simeq f_\nu^2 \frac{v^2}{M_R}$$

Minkowski (1977)
Yanagida (1979)
Gell-Mann, Ramond, Slansky (1980)
Mohapatra & Senjanovic (1980)

- For $f_\nu v \simeq 100$ GeV, $M_R \simeq (10^{14} - 10^{15})$ GeV.

Seesaw Mechanism in Matrix Form

- Including a Dirac mass term that connects ν_L and ν_R and the Majorana mass term for ν_R, the 2x2 neutrino mass matrix looks:

$$M_\nu = \begin{pmatrix} 0 & m_D \\ m_D & M_R \end{pmatrix}$$

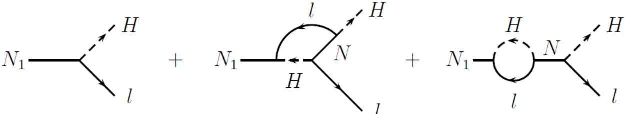
- This matrix has one heavy eigenvalue and one light eigenvalue when $m_D \ll M_R$:

$$\begin{aligned} M_N &= M_R \\ m_\nu &= \frac{m_D^2}{M_R} \end{aligned}$$

- As M_R becomes larger and larger, m_ν becomes smaller and smaller. Hence the name seesaw

Baryogenesis via leptogenesis and type-I seesaw

- In the early history of the universe, a lepton asymmetry may be dynamically generated in the decay of N Fukugita, Yanagida (1986)
- N being a Majorana fermion can decay to $L+H$ as well as $\overline{L}+H^*$



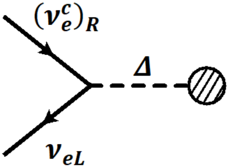
- Three Sakharov conditions can be satisfied: B violation via electroweak sphaleron, C and CP violation in Yukawa couplings of N, and out of equilibrium condition via expanding universe
- Lepton asymmetry in decay of N_1 (with $M_1 \ll M_{2,3}$):

$$\varepsilon_1 \simeq \frac{3}{16\pi} \frac{1}{(f_\nu f_\nu^\dagger)_{11}} \sum_{i=2,3} \mathrm{Im}\left[(f_\nu f_\nu^\dagger)_{i1}^2\right] \frac{M_1}{M_i}$$

- $\varepsilon \sim 10^{-6}$ can explain observed baryon asymmetry of the universe
- Indirect tests in Majorana nature of ν and in CP violation in oscillations

A Second Way of Seesawing Neutrino Mass

- Neutrino Majorana mass can arise even in the absence of ν_R. Simply use $(\nu^c)_R$ in its place:



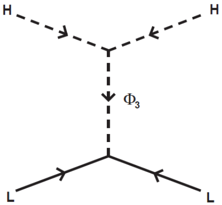
- Δ here is a new Higgs triplet field which has a nonzero vacuum expectation value that breaks Lepton Number. This expectation value is tiny compared to the usual Higgs expectation value, since the mass of Δ is very large!

- This way of generating small neutrino mass is called the Type-II seesaw mechanism

Seesaw mechanism (cont.)

Type II seesaw: $\Phi_3 \sim (1,3,1)$

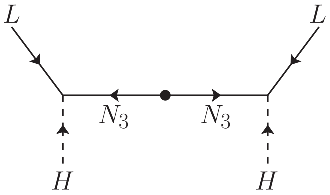
Mohapatra & Senjanovic (1980)
Schechter & Valle (1980)
Lazarides, Shafi, & Wetterich (1981)



Type III seesaw: $N_3 \sim (1,3,0)$

Foot, Lew, He, & Joshi (1989)

Ma (1998)



- Φ_3 abd N_3 contain charged particles which can be looked for at LHC
- Eg: $\Phi^{++} \to \ell^+\ell^+$, $\Phi^{++} \to W^+W^+$ decays would establish lepton number violation

Dirac Neutrino Mass Models

- Neutrinos may be Dirac particles without lepton number violation
- Oscillation experiments cannot distinguish Dirac neutrinos from Majorana neutrinos
- Spin-flip transition rates (in stars, early universe) are suppressed by small neutrino mass:

$$\Gamma_{\mathrm{spin-flip}} \approx \left(\frac{m_\nu}{E}\right)^2 \Gamma_{\mathrm{weak}}$$

- If neutrinos are Dirac, it would be nice to understand the smallness of their mass
- Models exist which explain the smallness of Dirac m_ν
- "Dirac leptogenesis" can explain baryon asymmetry

  Dick, Lindner, Ratz, Wright (2000)

Dirac Seesaw Models

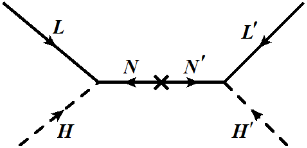
- Dirac seesaw can be achieved in Mirror Models Lee, Yang (1956); Foot, Volkas (1995); Berezhiani, Mohapatra (1995), Silagadze(1997)
  and Left-Right Symmetric Models Mohapatra (1988); Babu, He (1989); Babu, He, Su, Thapa (2022)

- Mirror sector is a replica of Standard Model, with new particles transforming under mirror gauge symmetry:

$$L = \begin{pmatrix} \nu \\ e \end{pmatrix}_L ; \quad H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix} ; \quad L' = \begin{pmatrix} \nu' \\ e' \end{pmatrix}_L ; \quad H' = \begin{pmatrix} H'^+ \\ H'^0 \end{pmatrix}$$

- Effective dimension-5 operator induces small Dirac mass:

$$\frac{(LH)(L'H')}{\Lambda} \Rightarrow m_\nu = \frac{vv'}{\Lambda}$$



Dirac Neutrino from Left-Right Symmetry

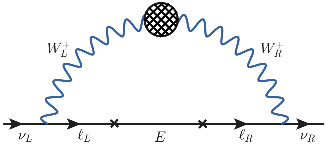
- Fermion transformation: $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$:

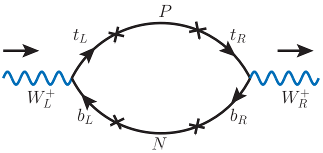
$$Q_L\ (3,2,1,1/3) = \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \quad Q_R\ (3,1,2,1/3) = \begin{pmatrix} u_R \\ d_R \end{pmatrix},$$

$$\Psi_L\ (1,2,1,-1) = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}, \quad \Psi_R\ (1,1,2,-1) = \begin{pmatrix} \nu_R \\ e_R \end{pmatrix}.$$

- Since neutrino Dirac mass arises via two-loop diagrams, it is extremely small

Babu, He (1989); Babu, He, Su, Thapa (2021)





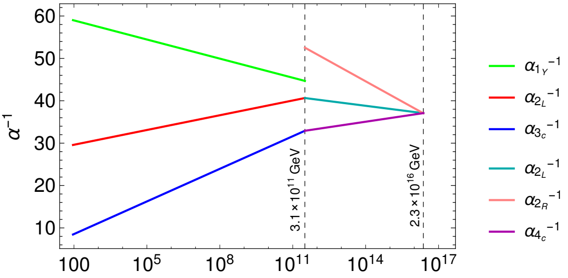
Unification of Forces & Matter

16 members of a family fit into a single unit (a spinor) in $SO(10)$

$u_r : \{-++\ +-\}$	$d_r : \{-++\ -+\}$	$u_r^c : \{+--\ ++\}$	$d_r^c : \{+--\ --\}$
$u_b : \{+-+\ +-\}$	$d_b : \{+-+\ -+\}$	$u_b^c : \{-+-\ ++\}$	$d_b^c : \{-+-\ --\}$
$u_g : \{++-\ +-\}$	$d_g : \{++-\ -+\}$	$u_g^c : \{--+\ ++\}$	$d_g^c : \{--+\ --\}$
$\nu : \{---\ +-\}$	$e : \{---\ -+\}$	$\nu^c : \{+++\ ++\}$	$e^c : \{+++\ --\}$

First 3 spins refer to color, last two are weak spins

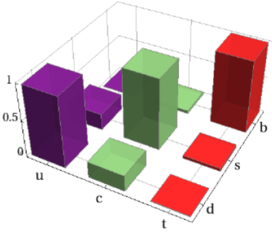
$$Y = \frac{1}{3}\Sigma(C) - \frac{1}{2}\Sigma(W)$$



Disparity in Quark & Lepton Mixings

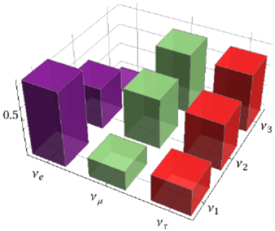
- **Quark Mixings**

$$V_{CKM} \sim \begin{bmatrix} 0.976 & 0.22 & 0.004 \\ -0.22 & 0.98 & 0.04 \\ 0.007 & -0.04 & 1 \end{bmatrix}$$



- **Leptonic Mixings**

$$U_{PMNS} \sim \begin{bmatrix} 0.85 & -0.54 & 0.16 \\ 0.33 & 0.62 & -0.72 \\ -0.40 & -0.59 & -0.70 \end{bmatrix}$$



Yukawa Sector of Minimal $SO(10)$

$$16 \times 16 = 10_s + 120_a + 126_s$$

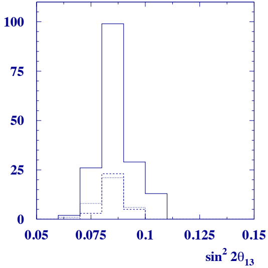
- At least two Higgs fields needed for family mixing
- Symmetric 10_H and $\overline{126}$ is the minimal model

$$W_{SO(10)} = 16^T \left(Y_{10}\, 10_H + Y_{126} \overline{126}_H \right) 16 \ .$$

$$\begin{aligned}
M_U &= v_u^{10} Y_{10} + v_u^{126} Y_{126} \\
M_D &= v_d^{10} Y_{10} + v_d^{126} Y_{126} \\
M_E &= v_d^{10} Y_{10} - 3 v_d^{126} Y_{126} \\
M_{\nu_D} &= v_u^{10} Y_{10} - 3 v_u^{126} Y_{126} \\
M_R &= Y_{126} V_R
\end{aligned}$$

Minimal Yukawa sector of SO(10)

- 12 parameters plus 7 phases to fit 18 observed quantities
- This setup fits all obsevables quite well
- Large neutrino mixings coexist with small quark mixings
- θ_{13} prediction turned out to be correct



Babu, Mohapatra (1993); Bajc, Senjanovic, Vissani (2001); (2003); Fukuyama, Okada (2002); Goh, Mohapatra, Ng (2003); Bajc, Melfo, Senjanovic, Vissani (2004); Bertolini, Malinsky, Schwetz (2006); Babu, Macesanu (2005); Dutta, Mimura, Mohapatra (2007); Aulakh et al (2004); Bajc, Dorsner, Nemevsek (2009); Joshipura, Patel (2011); Dueck, Rodejohann (2013); Ohlsson, Penrow (2019); Babu, Bajc, Saad (2018); Babu, Saad (2021)

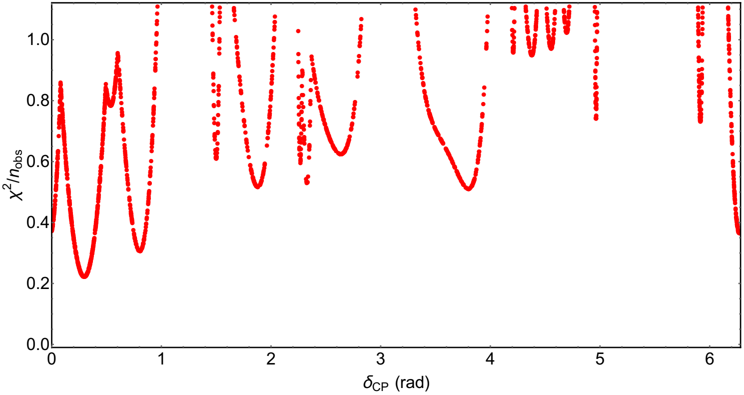
Best fit values for fermion masses and mixings

Observables (masses in GeV)	**SUSY**			**non-SUSY**		
	Input	Best Fit	Pull	Input	Best Fit	Pull
$m_u/10^{-3}$	0.502±0.155	0.515	0.08	0.442±0.149	0.462	0.13
m_c	0.245±0.007	0.246	0.14	0.238±0.007	0.239	0.18
m_t	90.28±0.89	90.26	-0.02	74.51±0.65	74.47	-0.05
$m_b/10^{-3}$	0.839±0.17	0.400	-2.61	1.14±0.22	0.542	-2.62
$m_s/10^{-3}$	16.62±0.90	16.53	-0.09	21.58±1.14	22.57	0.86
m_b	0.938±0.009	0.933	-0.55	0.994±0.009	0.995	0.19
$m_e/10^{-3}$	0.3440±0.0034	0.344	0.08	0.4707±0.0047	0.470	-0.03
$m_\mu/10^{-3}$	72.625±0.726	72.58	-0.05	99.365±0.993	99.12	-0.24
m_τ	1.2403±0.0124	1.247	0.57	1.6892±0.0168	1.688	-0.05
$\|V_{us}\|/10^{-2}$	22.54±0.07	22.54	0.02	22.54±0.06	22.54	0.06
$\|V_{cb}\|/10^{-2}$	3.93±0.06	3.908	-0.42	4.856±0.06	4.863	0.13
$\|V_{ub}\|/10^{-2}$	0.341±0.012	0.341	0.003	0.420±0.013	0.421	0.10
δ°_{CKM}	69.21±3.09	69.32	0.03	69.15±3.09	70.24	0.35
$\Delta m^2_{21}/10^{-5}(eV^2)$	8.982±0.25	8.972	-0.04	12.65±0.35	12.65	-0.01
$\Delta m^2_{31}/10^{-3}(eV^2)$	3.05±0.04	3.056	0.02	4.307±0.059	4.307	0.006
$\sin^2\theta_{12}$	0.318±0.016	0.314	-0.19	0.318±0.016	0.316	-0.07
$\sin^2\theta_{23}$	0.563±0.019	0.563	0.031	0.563±0.019	0.563	0.01
$\sin^2\theta_{13}$	0.0221±0.0006	0.0221	-0.003	0.0221±0.0006	0.0220	-0.16
δ°_{CP}	224.1±33.3	240.1	0.48	224.1±33.3	225.1	0.03
χ^2	-	-	7.98	-	-	7.96

Babu, Saad (2021)

Dirac CP phase

Multiple χ^2 minima make δ_{CP} prediction difficult



Babu, Bajc, Saad (2018)

Proton decay predictions

- Proton decay branching ratios determined by neutrino oscillation fits
- Mediated by superheavy gauge bosons
- Lifetime has large uncertainties, $\tau_p \approx (10^{32} - 10^{36})$ yrs.

Prediction of branching ratios

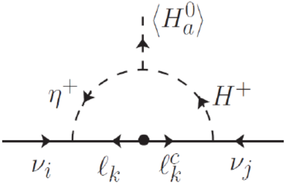
$$\Gamma(p \to \pi^0 e^+) \to 47\%$$
$$\Gamma(p \to \pi^0 \mu^+) \to 1\%$$
$$\Gamma(p \to \eta^0 e^+) \to 0.20\%$$
$$\Gamma(p \to \eta^0 \mu^+) \to 0.00\%$$
$$\Gamma(p \to K^0 e^+) \to 0.16\%$$
$$\Gamma(p \to K^0 \mu^+) \to 3.62\%$$
$$\Gamma(p \to \pi^+ \overline{\nu}) \to 48\%$$
$$\Gamma(p \to K^+ \overline{\nu}) \to 0.22\%$$

Nemesvek, Bajc, Dorsner (2009)
Babu, Khan (2015)

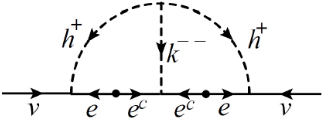
Radiative neutrino mass generation

- An alternative to seesaw is radiative neutrino mass generation, where neutrino mass is absent at tree level, but arises via quantum loop corrections
- The smallness of neutrino mass is explained by loop and chiral suppressions
- Loop diagrams may arise at 1-loop, 2-loop or 3-loop levels
- New physics scale typically near TeV and thus accessible to LHC
- Further tests in observable LFV processes and as nonstandard neutrino interaction (NSI) in oscillations

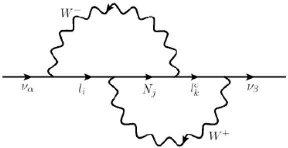
Radiative Neutrino Mass Models



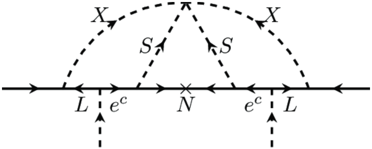
Zee



Zee-Babu



KNT



Effective $\Delta L = 2$ Operators

$$
\begin{aligned}
\mathcal{O}_1 &= L^i L^j H^k H^l \epsilon_{ik}\epsilon_{jl} \\
\mathcal{O}_2 &= L^i L^j L^k e^c H^l \epsilon_{ij}\epsilon_{kl} \\
\mathcal{O}_3 &= \{L^i L^j Q^k d^c H^l \epsilon_{ij}\epsilon_{kl}, \quad L^i L^j Q^k d^c H^l \epsilon_{ik}\epsilon_{jl}\} \\
\mathcal{O}_4 &= \{L^i L^j \bar{Q}_i \bar{u^c} H^k \epsilon_{jk}, \quad L^i L^j \bar{Q}_k \bar{u^c} H^k \epsilon_{ij}\} \\
\mathcal{O}_5 &= L^i L^j Q^k d^c H^l H^m \bar{H}_i \epsilon_{jl}\epsilon_{km} \\
\mathcal{O}_6 &= L^i L^j \bar{Q}_k \bar{u^c} H^l H^k \bar{H}_i \epsilon_{jl} \\
\mathcal{O}_7 &= L^i Q^j \bar{e^c} \bar{Q}_k H^k H^l H^m \epsilon_{il}\epsilon_{jm} \\
\mathcal{O}_8 &= L^i \bar{e^c} \bar{u^c} d^c H^j \epsilon_{ij} \\
\mathcal{O}_9 &= L^i L^j L^k e^c L^l e^c \epsilon_{ij}\epsilon_{kl} \\
\mathcal{O}_1' &= L^i L^j H^k H^l \epsilon_{ik}\epsilon_{jl} H^{*m} H_m
\end{aligned}
$$

Babu & Leung (2001)

de Gouvea & Jenkins (2008)

Angel & Volkas (2012)

Cai, Herrero-Garcia, Schmidt, Vicente, Volkas (2017)

Lehman (2014) – all $d = 7$ operators

Li, Ren, Xiao, Yu, Zheng (2020); Liao, Ma (2020) – all $d = 9$ operators

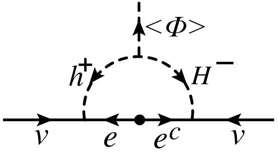
Operator $\mathcal{O}_2$ and the Zee model

- Introduce a singly charged scalar and a second Higgs doublet to standard model:

$$\mathcal{L} = f_{ij} L_i^a L_j^b h^+ \epsilon_{ab} + \mu H^a \Phi^b h^- \epsilon_{ab} + \text{h.c.}$$
$$\Downarrow$$
$$\mathcal{O}_2 = L^i L^j L^k e^c H^l \epsilon_{ij} \epsilon_{kl}$$

Zee (1980)

- Neutrino mass arises at one-loop.



- A minimal version of this model in which only one Higgs doublet couples to a given fermion sector with a Z_2 symmetry yields: Wolfenstein (1980)

$$m_\nu = \begin{pmatrix} 0 & m_{e\mu} & m_{e\tau} \\ m_{e\mu} & 0 & m_{\mu\tau} \\ m_{e\tau} & m_{\mu\tau} & 0 \end{pmatrix}, \quad m_{ij} \simeq \frac{f_{ij}}{16\pi^2} \frac{(m_i^2 - m_j^2)}{\Lambda}$$

It requires $\theta_{12} \simeq \pi/4 \rightarrow$ ruled out by solar + KamLAND data.

Koide (2001); Frampton *et al.* (2002); He (2004)

Neutrino oscillations in the Zee model

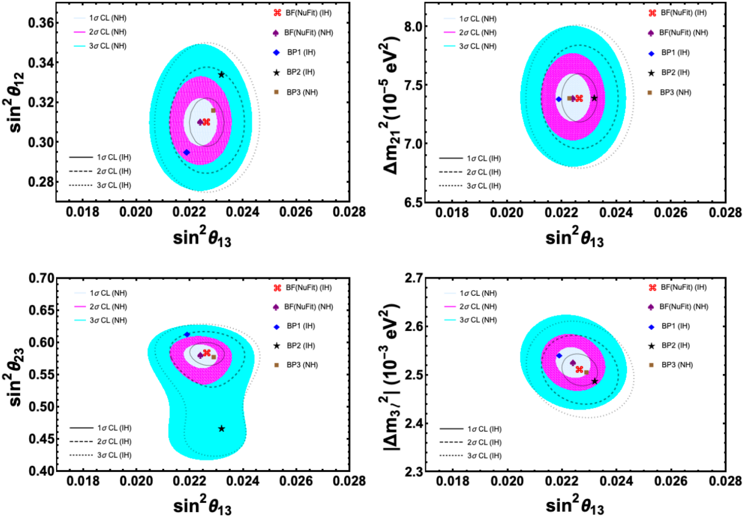
- Neutrino oscillation data can be fit to the Zee model consistently without the Z_2 symmetry

- Some benchmark points for Yukawa couplings of second doublet:

$$\begin{aligned}
\text{BP I}: \; Y &= \begin{pmatrix} Y_{ee} & 0 & Y_{e\tau} \\ 0 & Y_{\mu\mu} & Y_{\mu\tau} \\ 0 & Y_{\tau\mu} & Y_{\tau\tau} \end{pmatrix} \\
\text{BP II}: \; Y &= \begin{pmatrix} 0 & Y_{e\mu} & Y_{e\tau} \\ Y_{\mu e} & 0 & Y_{\mu\tau} \\ 0 & Y_{\tau\mu} & Y_{\tau\tau} \end{pmatrix} \\
\text{BP III}: \; Y &= \begin{pmatrix} Y_{ee} & 0 & Y_{e\tau} \\ 0 & Y_{\mu\mu} & Y_{\mu\tau} \\ Y_{\tau e} & 0 & Y_{\tau\tau} \end{pmatrix}
\end{aligned}$$

Babu, Dev, Jana, Thapa (2019)

Neutrino fit in the Zee model



Babu, Dev, Jana, Thapa (2019)

Symmetries of Neutrino Mass Matrix

- Neutrino mass matrix may have certain flavor symmetries that may constrain parameters of oscillations

- Such symmetries often lead to zeros in the neutrino mass matrix: texture zero

- Majorana neutrino mass matrix is symmetric, at most two texture zeros are admisssible, consistent with neutrino data

- Each case makes a prediction for currently unknown oscillation parameters: CP violation, lightest neutrino mass, mass ordering

Texture Zeros

$$A_1 : \begin{pmatrix} 0 & 0 & X \\ 0 & X & X \\ X & X & X \end{pmatrix} \qquad A_2 : \begin{pmatrix} 0 & X & 0 \\ X & X & X \\ 0 & X & X \end{pmatrix}$$

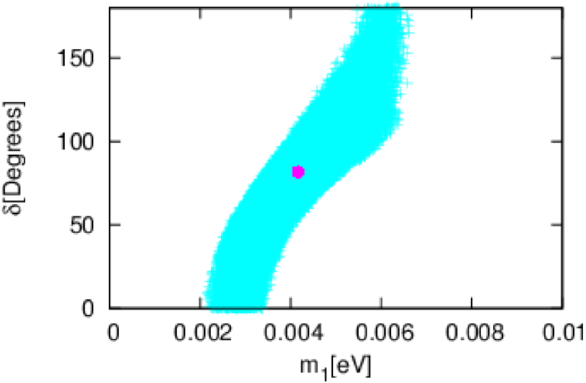
$$B_1 : \begin{pmatrix} X & X & 0 \\ X & 0 & X \\ 0 & X & X \end{pmatrix} \qquad B_2 : \begin{pmatrix} X & 0 & X \\ 0 & X & X \\ X & X & 0 \end{pmatrix}$$

$$B_3 : \begin{pmatrix} X & 0 & X \\ 0 & 0 & X \\ X & X & X \end{pmatrix} \qquad B_4 : \begin{pmatrix} X & X & 0 \\ X & X & X \\ 0 & X & 0 \end{pmatrix}$$

$$C : \begin{pmatrix} X & X & X \\ X & 0 & X \\ X & X & 0 \end{pmatrix}$$

Frampton, Glashow, Marfatia (2002)
Xing (2002)
Merle, Rodejohann (2006)
Goswami et. al (2006)

Texture Zero Predictions



Conclusions

- Neutrino may be either a Dirac fermion or a Majorana fermion
- Observation of neutrinoless double beta decay will confirm its Majorana nature
- Seesaw mechanism very attractive framework to explain smallness of Majorana neutrino masses
- Unified theories can be very predictive for neutrino neutrino masses and mixings
- Flavor symmetries may play a role, which would predict certain oscillation parameters
- Further experiments needed to probe deep into the origin of neutrino mass!